Connectedness

A topological space (X, τ) is *disconnected* if any one of the following equivalent properties holds:

1. X can be written as the union of two disjoint, nonempty, open subsets.
2. X can be written as the union of two disjoint, nonempty, closed subsets.
3. X has a proper, clopen subset.

A topological space (X, τ) is *connected* if X is not disconnected!

Examples

Are the following topological spaces connected?

(a) $\mathbb{N}$ with the discrete topology

(b) $\mathbb{N}$ with the indiscrete topology

(c) $\mathbb{N}$ with the initial segment topology

(d) $\mathbb{N}$ with the final segment topology

(e) $\mathbb{N}$ with the finite-closed topology

(f) $\mathbb{R}$ with the Euclidean topology

(g) $\mathbb{R}$ with the Sorgenfrey topology

(h) the Cartesian product of two disconnected spaces, with the product topology

(i) the Cartesian product of two connected spaces, with the product topology

Assignment due next class

1. Write a solution to number 8 in Exercises 3.2 (about the interaction of interior and closure with union and intersection).
2. Write a solution to number 6 in Exercises 3.3 (about connectedness and the countable-closed topology).
3. Read section 4.1 in the textbook (about the subspace topology).