Subspace topology

If (X, τ) is a topological space, and Y is a sub**set** of X, then Y is called a sub**space** when Y is equipped with the topology τ_Y consisting of all sets $U \cap Y$, where $U \in \tau$.

Warning: The words "open" and "closed" have different meanings in X and in Y.

Example

Suppose $X = \mathbb{R}$ with the Euclidean topology, and $Y = \mathbb{Q}$ with the subspace topology. Let A be $\{ x \in \mathbb{Q} : 0 < x < 1 \}$.
The set A is "open in Y" (meaning $A \in \tau_Y$) but not "open in X" ($A \notin \tau$).
Relative to X, the interior of A is empty. Relative to Y, the interior of A is A. The closure of A in $\mathbb{R}$ is $[0, 1]$. The closure relative to Y is $\{ x \in \mathbb{Q} : 0 \leq x \leq 1 \}$.

Consistency questions

1. If Y is a subspace of X, and A is a *closed* subset of X, is $A \cap Y$ a relatively closed subset of Y?
   Yes.

Assignment due next class

1. Write solutions to numbers 6 and 11 in Exercises 4.1.
2. Read section 4.2 in the textbook.