Reminder

- The second exam takes place on April 11 (next Wednesday).
- The material for the exam is mainly Chapter 5 and Sections 6.1 and 6.2.

A new generalization of the Euclidean topology on $\mathbb{R}$

Suppose X is a set equipped with a strict linear order relation "$<$": namely,

- if $x < y$ and $y < z$, then $x < z$ (transitivity),
- it never happens that $x < x$ (irreflexivity),
- for every two distinct points x and y, either $x < y$ or $y < x$ (but not both).

A subbasis for the *order topology* on X consists of sets of the form $\{x \in X : a < x\}$ and $\{x \in X : x < b\}$ for arbitrary points a and b in X. A basis consists of these sets together with sets of the form $\{x \in X : a < x < b\}$.

Examples of the order topology

1. If $X = \mathbb{R}$ and $<$ is the usual inequality relation, then the order topology agrees with the standard Euclidean topology.
2. If $X = (0,1) \cup \{3\}$, then the order topology on X is different from the subspace topology.

   The subset $\{3\}$ equals $X \cap (2,4)$, so is open in the subspace topology.

   In the order topology, a neighborhood of the point 3 has to contain some set $\{x \in X : a < x\}$ for which $a < 1$: every neighborhood of 3 must intersect $(0,1)$. So $\{3\}$ is not an open set in the order topology on X.

Review exercise

Suppose $X = (0,1) \cup \{3\}$.

- For each of the following topologies, is the space X connected? If not, what are the components of X?
- Same question for path-connected and path-components.

1. discrete topology
2. indiscrete topology
3. the subspace topology induced by $(\mathbb{R}, \text{Euclidean})$
4. the subspace topology induced by $(\mathbb{R}, \text{Sorgenfrey})$
5. the finite-closed topology
6. the order topology

Assignment due next class

Study for the upcoming exam.