Announcements/reminders

- Class meets Monday (April 30) and Tuesday (May 1).
- I will hold my usual office hour 3:00–4:00 in the afternoon on Tuesday (May 1) and Thursday (May 3).
- The comprehensive final examination takes place 10:30–12:30 on Friday (May 4).
- Material for the final exam: Chapters 1–5, Sections 6.1–6.2, and Chapter 7.

Quotient spaces (identification spaces): an example

Suppose $X = \mathbb{N}$. Consider the *partition* of X into the set of even numbers and the set of odd numbers.

Identifying all the even numbers as a single point and all the odd numbers as a single point produces a new space Y, which could be written as $\{E, O\}$.

A topology τ on X induces a natural topology on Y: namely, $\varnothing$ and Y must be open sets, and $\{E\}$ is declared to be open if and only if the set of even numbers belongs to τ, and $\{O\}$ is open if and only if the set of odd numbers belongs to τ.

So if X has the finite-closed topology, then Y has the indiscrete topology. If X has the discrete topology, so does Y.

Another example

Let X be $\mathbb{R}$ with the Euclidean topology.

Define an equivalence relation on X by saying that x_1 is related to x_2 when $x_1 - x_2$ is an integer. Partition X into equivalence classes.

One equivalence class is $\mathbb{Z}$. Every other equivalence class can be expressed as a set $x + \mathbb{Z}$, where x is some real number between 0 and 1.

Let Y be the set of equivalence classes. Define $f : X \to Y$ to be the function that sends x to the equivalence class of x. And define a subset U of Y to be open in the quotient topology when $f^{-1}(U)$ is open in X.

This quotient space Y is homeomorphic to a circle.

A practice/review exercise (not to hand in)

Solve number 5 in Exercises 7.2, which is also Remark 11.1.8.