$$\int_0^\infty \frac{\sqrt{x}}{(x^2+1)^2}\,dx = \frac{\pi}{4\sqrt{2}}$$

To be revisited later.

$$\frac{1}{\pi}\int_{-1}^{1}\frac{\sqrt{1-t^2}}{z-t}\,dt = z-\sqrt{z^2-1}$$

Replace z by $-z$ to get

$$\frac{1}{\pi}\int_{-1}^{1}\frac{\sqrt{1-t^2}}{-z-t}\,dt = -\frac{1}{\pi}\int_{-1}^{1}\frac{\sqrt{1-t^2}}{z+t}\,dt$$

change variables

$$-\frac{1}{\pi}\int_{-1}^{1}\frac{\sqrt{1-t^2}}{z-t}\,dt$$

Exercise. Represent complex numbers z and w by vectors (x, y) and (u, v) in $\mathbb{R}^2$. How is the complex number $\overline{z}w$ related to the usual dot product and cross product of the vectors (x, y) and (u, v)?

Exercise 1 on page 4.

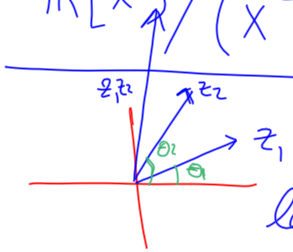
$a+bi \quad , \quad a \in \mathbb{R}, \ b \in \mathbb{R}$

$i^2 = -1$

$(a,b)\cdot(c,d)$

$:= (ac-bd \ , ad+bc)$

$\mathbb{R}[x]/(x^2+1) = \mathbb{C}$



angle of z_1z_2

$= \theta_1 + \theta_2$

length of z_1z_2

$= (\text{length } z_1)(\text{length } z_2)$

$a+bi \sim (a,b)$

$\sim \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

---

length of z

$= |z|$ absolute value or modulus

$z = x+iy \qquad \bar{z} = x-iy$

$x = \mathrm{Re}(z) \qquad y = \mathrm{Im}(z)$