

## Examination 1

**Instructions:** Please write your solutions on your own paper. These problems should be treated as essay questions to answer in complete sentences.

1. Find all values of the complex variable  $z$  for which

$$(\operatorname{Re}(z))^4 = \operatorname{Re}(z^4).$$

2. When the letter  $z$  represents a complex variable, is it valid to say that  $\lim_{z \rightarrow \infty} (z + \bar{z}) = \infty$ ? Explain why or why not.
3. Suppose  $f$  is an analytic function on  $\mathbb{C} \setminus \{0\}$ , and the real part of  $f(z)$  equals  $\frac{\sin(2\theta)}{r^2}$  in standard polar coordinates. (As usual,  $r = |z|$ , and  $\theta = \arg(z)$ .) Find a concrete expression for  $f$  as a function of the variable  $z$ .
4. Suppose that  $c_n$  is a complex number for each natural number  $n$ , and the power series  $\sum_{n=1}^{\infty} c_n z^n$  has radius of convergence equal to 4. What can you say about the radius of convergence of the power series  $\sum_{n=1}^{\infty} c_n^2 z^n$ ? Explain how you know.
5. Suppose  $r$  is a positive real number, and  $\gamma_r(t) = re^{it}$  when  $0 \leq t \leq \pi$ . (This path is a half circle in the upper half-plane.) Is the path integral  $\int_{\gamma_r} \frac{1}{z} dz$  independent of the value of  $r$ ? Explain why or why not.
6. The diagram shows a mapping of a square by some analytic function  $f$ . (The dashed lines represent the coordinate axes.) Assuming that the value of  $a$  is chosen suitably, can  $f(z)$  be equal to  $1/z$ ? or  $z^2$ ? or  $e^z$ ? or must  $f(z)$  be something else? Explain how you know.

