

Math 650-600: Several Complex Variables

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Exercises on the Levi form

The Levi form is

$$\sum_{j,k=1}^n \frac{\partial^2 \rho}{\partial z_j \partial \bar{z}_k} t_j \bar{t}_k \quad \text{on complex tangent vectors } t,$$

where ρ is a defining function for a domain.

1. Positivity of the Levi form is independent of the choice of defining function.
2. Positivity of the Levi form is invariant under holomorphic changes of coordinates.

Consequences of a negative eigenvalue

Suppose 0 is a boundary point of Ω . Consider the Taylor expansion of a defining function ρ at 0. A suitable local biholomorphic change of variables reduces the expansion to:

$$\rho(z) = 2 \operatorname{Re} z_n + \sum_{j,k=1}^n \frac{\partial^2 \rho}{\partial z_j \partial \bar{z}_k}(0) z_j \bar{z}_k + o(|z|^2).$$

to be continued ...