Cauchy integral for disk

$$f(w) = \frac{1}{2\pi i} \int_{|z|=1} \frac{f(z)}{z-w}\, dz$$

Equivalent form

$$f(w) = \frac{1}{\pi} \int_{|z|<1} \frac{f(z)}{(1-w\bar{z})^2}\, dx\, dy,$$

Bergman kernel function for disk

$$K(w,z) = \frac{1}{\pi(1-w\bar{z})^2}$$

Bergman kernel of a domain

$$K(w, z) = \sum_j \phi_j(w)\overline{\phi}_j(z),$$

where $\{\phi_j\}$ is an orthonormal basis for the square-integrable holomorphic functions.

In the class of holomorphic functions f such that $f(z) \geq \int |f|^2$, the one with the maximal value at z is $K(w, z)$.

The Riemann map f is related to the Bergman kernel function K via

$$f'(z) = K(z,a)\sqrt{\frac{\pi}{K(a,a)}}.$$

The Bergman metric

$$g_{jk}(z) = \frac{\partial^2}{\partial z_j \partial \bar{z}_k} \log K(z,z).$$

Bergman Prize winners

David Catlin (1989)

(*Princeton*, Chicago, *Princeton*, Purdue)

Steve Bell and Ewa Ligocka (1991)

(MIT, *Princeton*, Purdue; Warsaw, Polish Acad. Sci.)

Charles Fefferman (1992)

(*Princeton*, Chicago, *Princeton*)

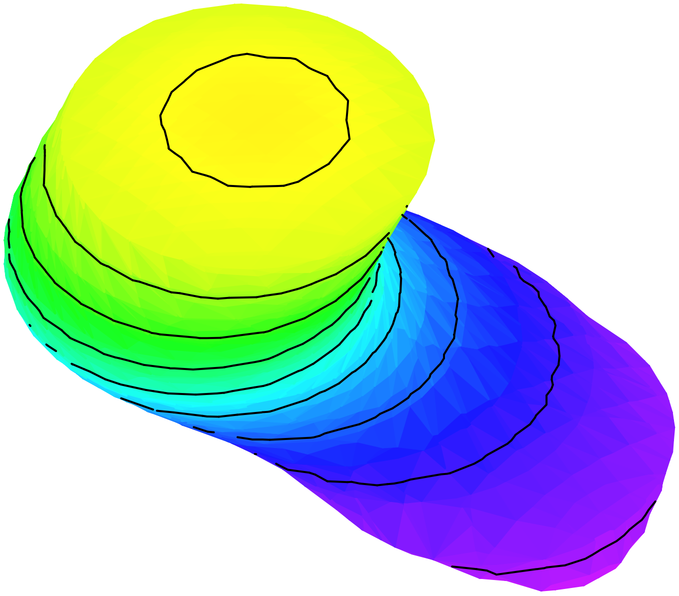
Yum-Tong Siu (1993)

(*Princeton*, Purdue, Notre Dame, Yale, Stanford, Harvard)

John Erik Fornæss (1994)

(Washington, *Princeton*, Michigan)

Diederich-Fornæss worm domain



A smoothly bounded pseudoconvex domain lacking a Stein neighborhood basis.

Theorem (Fefferman). *Let $f : D \to G$ be a biholomorphic mapping between bounded strongly pseudoconvex domains in $\mathbf{C}^n$ with boundaries of class C^∞. Then f extends to a diffeomorphism of the closures.*

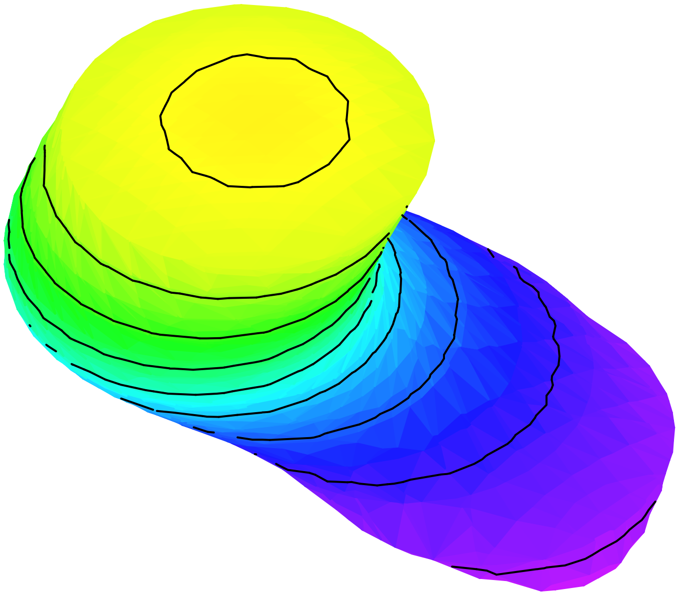
Theorem (Bell and Ligocka). *Let $f : D \to G$ be a biholomorphic mapping between bounded domains in $\mathbf{C}^n$ with boundaries of class C^∞, and suppose the Bergman projections preserve the space of functions that are C^∞ on the closure. Then f extends to a diffeomorphism of the closures.*

Theorem (Catlin). *A necessary and sufficient condition for the* $\overline{\partial}$*-Neumann problem to be subelliptic on a (weakly) pseudoconvex domain is that the boundary have bounded order of contact with complex varieties.*

Theorem (Boas and Straube). *The Fefferman mapping theorem holds for all (weakly) convex domains.*

More generally, the $\overline{\partial}$*-Neumann problem is regular in both* C^∞ *and each Sobolev space* W^k *for domains that are defined by plurisubharmonic functions.*

Theorem (Michael Christ, 1995). *The $\overline{\partial}$-Neumann operator and the Bergman projection are not C^∞ globally regular in the Diederich-Fornæss worm domain.*



We are dealing with a field of applied mathematics where the interaction between mathematics and physics is highly stimulating and fruitful for both sciences. – Bergman and Schiffer, 1953

... with the aid of modern computational technique, these processes come well within the range of practical application. – Bergman, 1950

Punch-card machine methods applied to the solution of the torsion problem, *Quart. Appl. Math.* **5** (1943), 69–81.